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EXERCISES.

396

SHOW how to solve the simultaneous equations:

1) $$x = y \sin(z + a) = y \sin z - a = y \sin(z + \beta) - b\,;$$

2) $$x(1 - \sin y) = a\,, \quad x[1 - \sin(y + \beta)] = b\,.$$

[*W. M. Thornton.*]

397

GIVEN two straight lines referred to rectangular coordinates; find geometrically an abscissa such that the sum of the squares of the two corresponding ordinates shall be a minimum. [*R. A. Harris.*]

398

IF any curve, plane or gauche, be represented by means of the coordinates P, Q, which are so connected with a set of orthogonal and isothermal coordinates p, q that $P =$ function p, $Q =$ function q, then the angles made by this curve and the curves, $Q =$ constant, $P =$ constant, are

$$\tan^{-1}\frac{dq}{dp}\,, \quad \tan^{-1}\frac{dp}{dq}\,,$$

respectively. [*R. A. Harris.*]

399

IF y be any cubic function of x between the limits 0, h, its mean value can be expressed in an infinite number of ways by the formula

$$\tfrac{1}{6}(y_1 - 2y_2 + y_3)(1 + 2\sin^2\varphi) + \tfrac{1}{2}(y_3 - y_1)\sin\varphi + y_2\,,$$

where

$$x_2 = \frac{h}{\sqrt{2}}\sec\varphi \sin(45° - \varphi)\,, \quad x_3 = x_2 + \tfrac{1}{2}h\sec\varphi\,, \quad x_1 = x_2 - \tfrac{1}{2}h\sec\varphi\,.$$

[*W. H. Echols.*]

400

IF y be any cubic function of x, its mean value in any interval $X_2 - X_1 = L$ can be expressed in an infinite number of ways in terms of only two ordinates by the formula

$$\frac{3\sin^2\varphi}{1 + 2\sin^2\varphi}\,y_3 + \left[1 - \frac{3\sin^2\varphi}{1 + 2\sin^2\varphi}\right]y_2 + \tfrac{1}{18}L^3\,\frac{1 - 4\sin^2\varphi}{4\sin^2\varphi}\tan\varphi\,,$$

where

$$x_2 = X_1 + \frac{1}{\sqrt{2}}L\sec\varphi\sin(45° - \varphi)\,, \quad x_3 = x_2 + \tfrac{1}{2}L\sec\varphi\,.$$

[*W. H. Echols.*]

401

DETERMINE the forms of surfaces which are such that the areas of parallel sections are cubic functions of their distances from a given section.

[*W. H. Echols.*]

402

TWO circular arcs are tangent to each other and to the sides a and b of a triangle ABC at the points A and B. Show that the difference of curvature of the arcs is least when their common tangent makes with a and b the angles $\frac{1}{2}(3B - A)$ and $\frac{1}{2}(3A - B)$, respectively. [*W. H. Echols.*]

403

IF a triangle be inscribed in an ellipse, by drawing chords parallel to the tangents at the points α, β, γ; prove that the sides are

$$c' = 2b' \sin(\gamma - \beta), \quad c'' = 2b'' \sin(\alpha - \gamma), \quad c''' = 2b''' \sin(\beta - \alpha),$$

where b', b'', b''' are the semi-diameters conjugate to those at the points α, β, γ. Also the area of the triangle $= 2ab \sin(\alpha - \gamma) \sin(\beta - \alpha) \sin(\gamma - \beta)$.

[*G. B. M. Zerr.*]

404

THE area of a rectangle circumscribing an ellipse is

$$\frac{4ab}{b'^2}\left[(a^2 + b^2)\, b'^2 - a^2 b^2\right]^{\frac{1}{2}},$$

where b' is the semi-diameter conjugate to that at the point of tangency of one of the sides. [*G. B. M. Zerr.*]

405

A TRAPEZOID circumscribing an ellipse is formed by the tangents at the extremities, and the tangents parallel to a chord passing through the points whose eccentric angles are α and β. Prove that its area $= 2ab \tan\theta \operatorname{cosec}^2\theta$, where $\theta = \frac{1}{4}(\alpha - \beta)$. [*G. B. M. Zerr.*]

406

IF y be any cubic function of x the mean value of y in any interval L is*

$$y_m + \frac{L^2}{24d^2}(y_1 + y_2 - 2y_m),$$

wherein y_m is the ordinate at the middle of the interval and the ordinates y_1 and y_2 are at a distance d on either side of y_m.

If x_m be the abscissa of the middle point of the interval, then in the limit when d vanishes we also have for the mean ordinate of the cubic $y = f(x)$,

$$y_m + \frac{L^2}{24} f''(x_m). \qquad [W.\ H.\ Echols.]$$

* A generalization of Newton's rule.